

UNIVERSITY OF BAHRAIN  
COLLEGE OF INFORMATION TECHNOLOGY  
DEPARTMENT OF COMPUTER SCIENCE  
ITCS251/252 DISCRETE MATHEMATICS  
SECOND SEMESTER 2011/2012

FIRST EXAM — 1 HOUR.

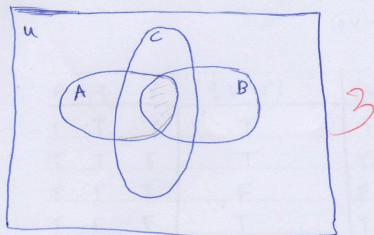
|              |               |
|--------------|---------------|
| STUDENT NAME | <b>DRAGON</b> |
| STUDENT#     |               |
| SECTION      |               |

| QUESTION# | MARKS             |    | REMARKS |
|-----------|-------------------|----|---------|
| 1         | 7                 | 7  |         |
| 2         | <del>6</del><br>8 | 6  |         |
| 3         | 6                 | 5  |         |
| 4         | 6                 | 6  |         |
| 5         | 8                 | 8  |         |
| TOTAL     | 35                | 32 |         |

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Dr. Yousif Harrath.

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Q1. (a) [3 points] Draw a Venn diagram showing that  $A \cap B \subset A \cap C$  but  $B \not\subset C$ .



$$A_1 = \{x \in \mathbb{Z} \mid x < -1 \text{ or } x > 1\} = \{\dots, -3, -2, 2, 3, \dots\}$$

(b) [2 points  $\times$  2] Let  $A_i = \{x \in \mathbb{Z} \mid x < -i \text{ or } x > i\}$ . Find  $A_2 = \{x \in \mathbb{Z} \mid x < -2 \text{ or } x > 2\} = \{\dots, -4, -3, 3, 4, \dots\}$

$$\overline{A_1} - \{0\} = \{-1, 0, 1\} - \{0\} = \{-1, 1\} \quad \text{and} \quad |\overline{A_2}| = |\{-2, -1, 0, 1, 2\}| = 5$$

$$\mathcal{P}(\overline{A_1}) = \mathcal{P}(\{-1, 0, 1\}) = \{\emptyset, \{-1, 0, 1\}, \{-1\}, \{0\}, \{1\}, \{-1, 0\}, \{-1, 1\}, \{0, 1\}\}$$

Q2. (a) Consider the following proposition:

If we are not in vacation, then we go to the university and study logic.

(1) [2 points] Write the proposition in symbolic form using only  $\neg, \wedge, \vee$ .

Let  $p$  = we are not in vacation,  $q$  = we go to the university,  $r$  = study logic

$$p \rightarrow (q \wedge r)$$

$$\text{in symbolic: } \equiv \sim p \vee (q \wedge r)$$

(2) [2 points] Write the negation of the proposition in symbolic form.

$$\sim (\sim p \vee (q \wedge r)) \equiv p \wedge \sim (q \wedge r)$$

$$\equiv p \wedge (\sim q \vee \sim r)$$

(3) [2 points] Write the contrapositive of the proposition in English.  $\sim (q \wedge r) \rightarrow \sim p \equiv (\sim q \vee \sim r) \rightarrow \sim p$

If we don't go to the university or we don't study logic, then we are in vacation.

(b) [2 points] Write the following proposition using "necessary condition".

It is not the case that if Hasan plays football and basketball, then he is not in his office or he is at home.

$$(P \wedge Q) \rightarrow (\neg R \vee S)$$

Hasan not in his office or he is at home are necessary conditions for Hasan to play football and basketball.



Q3. [6 points] Write the truth table for the following proposition.

5

$$(q \vee \neg r) \leftrightarrow (\neg r \rightarrow \neg p)$$

| $p$ | $q$ | $r$ | $(q \vee \neg r)$ | $(\neg r \rightarrow \neg p)$ | $(q \vee \neg r) \leftrightarrow (\neg r \rightarrow \neg p)$ |
|-----|-----|-----|-------------------|-------------------------------|---|
| T   | T   | T   | T                 | $F \rightarrow F = T$         | T   |
| T   | T   | F   | T                 | $T \rightarrow F = F$         | F   |
| T   | F   | T   | F                 | $F \rightarrow T = T$         | F   |
| T   | F   | F   | T                 | $T \rightarrow F = F$         | <del>T</del> F  |
| F   | T   | T   | T                 | $F \rightarrow F = T$         | T   |
| F   | T   | F   | T                 | $T \rightarrow F = F$         | <del>F</del> T  |
| F   | F   | T   | F                 | $F \rightarrow T = T$         | F   |
| F   | F   | F   | T                 | $T \rightarrow T = T$         | T   |

2pts
1.5
1.5

Q4. Show whether the following propositions are true or false. Justify your answer without using truth tables.

(1) [2 points] Let  $r$  be always true,  $p \vee q \equiv p \vee q \wedge r$ .

if  $r$  always true

$$p \vee q \wedge r \equiv p \vee q \wedge T \equiv p \vee q$$

$$\therefore p \vee q \equiv p \vee q \wedge r$$

(2) [2 points]  $\neg p \rightarrow \neg q$  is logically equivalent to  $p \rightarrow q$ .

$$\neg p \rightarrow \neg q \equiv p \vee \neg q$$

$$p \rightarrow q \equiv \neg p \vee q$$

$$\therefore p \vee \neg q \neq \neg p \vee q$$

$\therefore \neg p \rightarrow \neg q$  is not logically equivalent to  $p \rightarrow q$

(3) [2 points]  $(p \wedge q) \vee \neg p \vee \neg q \equiv T$ , where  $T$  is a tautology.

$$\neg p \vee \neg q \equiv \neg(p \wedge q)$$

$$(p \wedge q) \vee \neg(p \wedge q)$$

$$T \vee F \equiv T$$

$$F \vee T \equiv T$$

$$\therefore (p \wedge q) \vee \neg p \vee \neg q \equiv T$$

Q5. [8 points] Use inference rules to prove that the following argument is valid.

$$(1) p \rightarrow \neg q$$

$$(2) \neg r \wedge \neg s \equiv \sim(r \vee s)$$

$$(3) \neg q \rightarrow (r \vee s)$$

$$\therefore \neg p \vee t$$

from (1) and (2)

$$p \rightarrow \neg q$$

$$\neg q \rightarrow (r \vee s)$$

$$(4) \therefore p \rightarrow (r \vee s) \quad \checkmark$$

from (4) and (2)

$$p \rightarrow (r \vee s) \quad \checkmark$$

$$\sim(r \vee s)$$

(8)

$$(5) \therefore \sim p$$

from (5):

$$\sim p \quad \checkmark$$

$$\therefore \neg p \vee t$$

$\therefore$  the argument is valid.